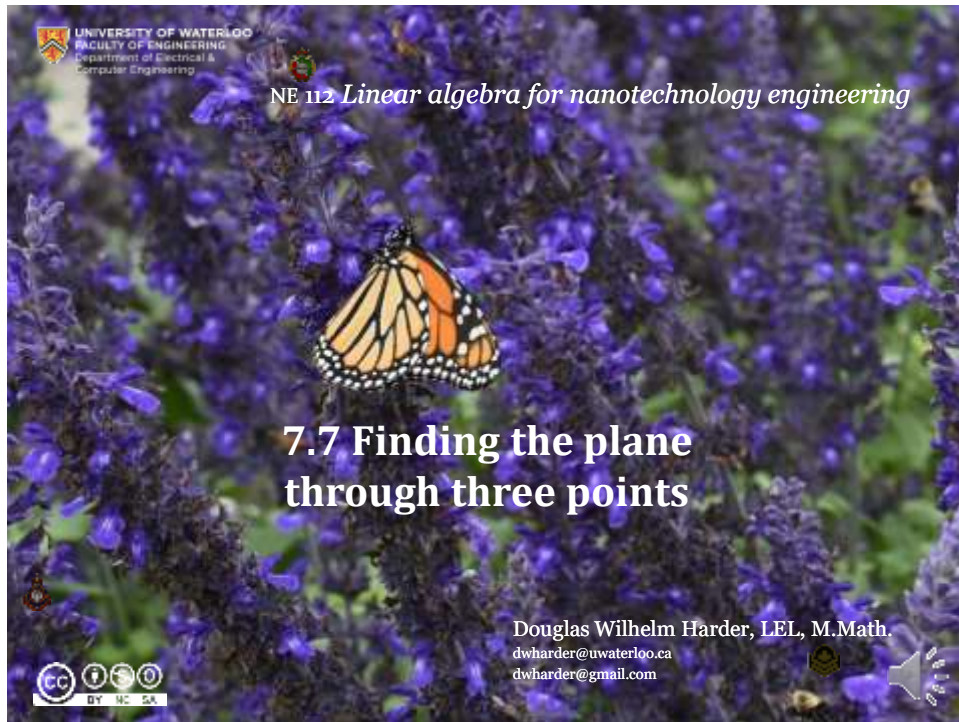
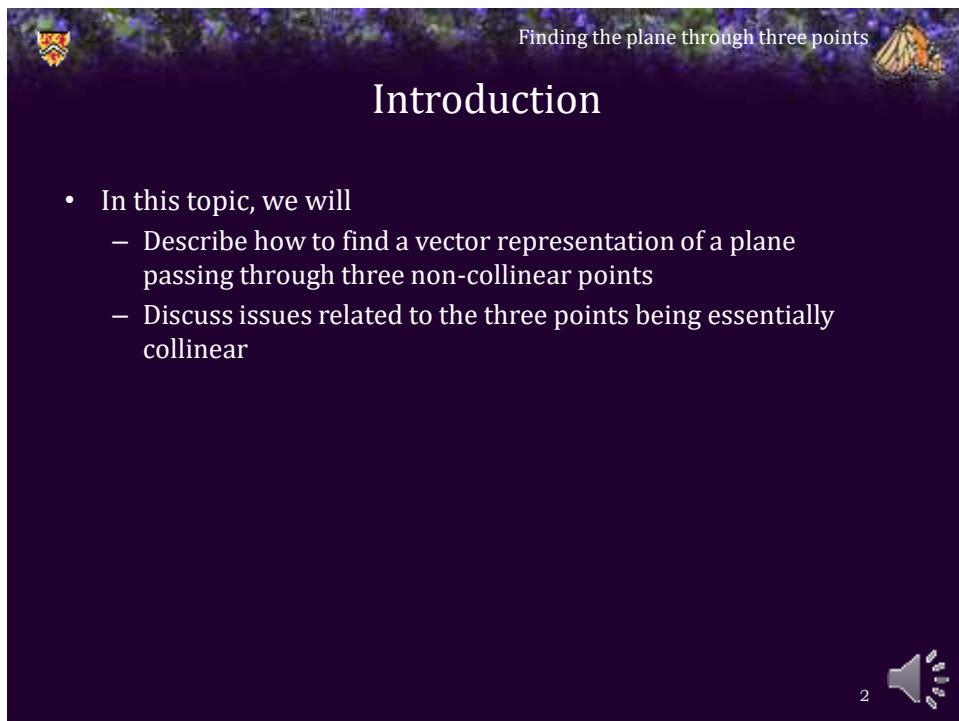




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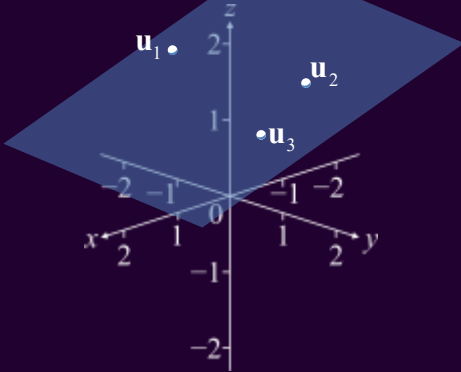


2

Finding the plane through three points

A plane through three points

- In $\mathbb{R}^3$, three points may be collinear or not
 - If they are collinear, they define a line in 3-space
 - If they are not collinear, they define a plane



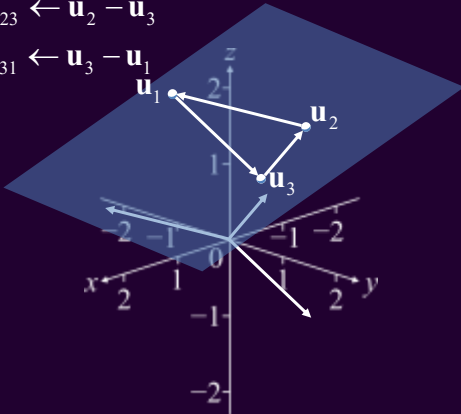
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Finding the plane through three points

A plane through three points

- Suppose we have three non-collinear vectors $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$
 - Define $\mathbf{v}_{12} \leftarrow \mathbf{u}_1 - \mathbf{u}_2$
 - $\mathbf{v}_{23} \leftarrow \mathbf{u}_2 - \mathbf{u}_3$
 - $\mathbf{v}_{31} \leftarrow \mathbf{u}_3 - \mathbf{u}_1$



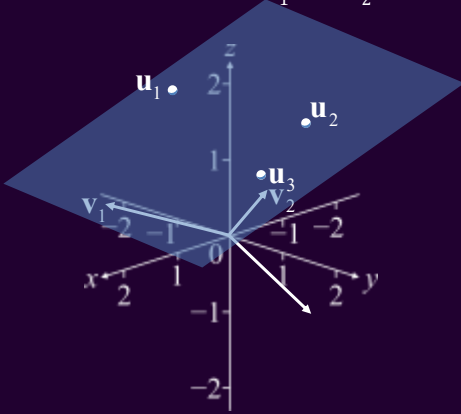
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Finding the plane through three points

A plane through three points

- All three of these vectors are parallel to the plane
 - Choose two vectors that have an angle between them closest to 90° and relabel them $\mathbf{v}_1$ and $\mathbf{v}_2$



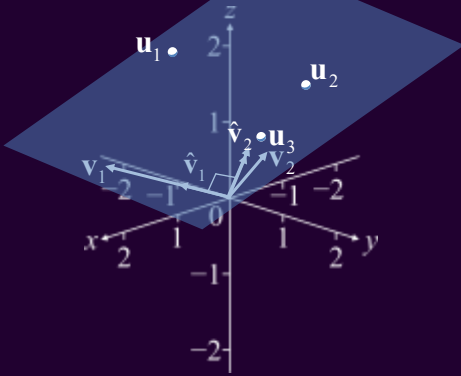
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Finding the plane through three points

A plane through three points

- Given the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$,
apply Gram-Schmidt resulting in two orthonormal vectors
parallel to the plane: $\hat{\mathbf{v}}_1$ and $\hat{\mathbf{v}}_2$



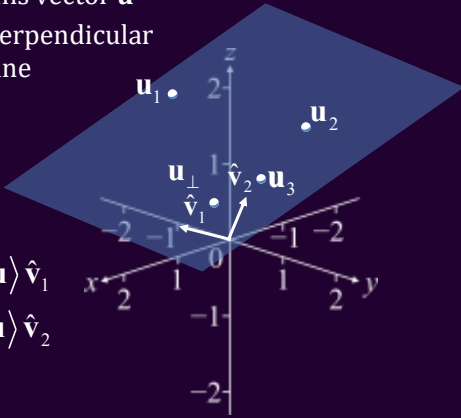
6

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Finding the plane through three points

A plane through three points

- All three vectors $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$ are in the plane, so choose that vector that has the smallest 2-norm
 - Relabel this vector $\mathbf{u}$
 - Make $\mathbf{u}$ perpendicular to the plane



$$\mathbf{u} \leftarrow \mathbf{u} - \langle \hat{\mathbf{v}}_1, \mathbf{u} \rangle \hat{\mathbf{v}}_1$$

$$\mathbf{u}_\perp \leftarrow \mathbf{u} - \langle \hat{\mathbf{v}}_2, \mathbf{u} \rangle \hat{\mathbf{v}}_2$$

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Finding the plane through three points

A plane through three points

- Thus, a vector representation of the plane is

$$\mathbf{u}_\perp + \alpha \hat{\mathbf{v}}_1 + \beta \hat{\mathbf{v}}_2$$
 - We can use $\mathbf{u}_\perp$ to find an equation of the plane

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Finding the plane through three points

Almost collinear

- Suppose we have three points that are not collinear, but that are very close to being collinear
 - For example, $\|\mathbf{v}_2\|_2 \ll \|\mathbf{v}_1\|_2$ would suggest two points are very close to each other
 - Issue: $\mathbf{v}_1$ and $\mathbf{v}_2$ may be almost parallel
 - We could calculate the cross product of these two vectors, and if $\|\mathbf{v}_1 \times \mathbf{v}_2\|_2 \ll \|\mathbf{v}_1\|_2 \|\mathbf{v}_2\|_2$
 - In either case, this will result in a high likelihood of any error being magnified, so it would be appropriate to declare the points *essentially* collinear and try to find a different point in the plane

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Finding the plane through three points

Summary

- Following this topic, you now
 - Know how to find a plane passing through three points
 - Understand that while three points may not be exactly collinear, they may be very close to being collinear

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Finding the plane through three points

References

[1] [https://en.wikipedia.org/wiki/Plane_\(geometry\)](https://en.wikipedia.org/wiki/Plane_(geometry))

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Finding the plane through three points

Acknowledgments

None so far.

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Finding the plane through three points

Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas. Mathematical equations are prepared in MathType by Design Science, Inc. Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see <https://www.rbg.ca/> for more information.



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Finding the plane through three points

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